International Journal of Engineering& Scientific Research
Vol.8 Issue 12, December 2020,
ISSN: 2347-6532 Impact Factor: 6.660
Journal Homepage: http://www.ijmra.us, Email: editorijmie@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

Proof behind rules of divisibility of some primes number

By

N.S.Bayaskar

Department of Mathematics Arts, Science and Commerce College Chikhaldara,Dist. Amravati (M.S.)

Abstract.

Divisibility rule provide simple techniques for determining whether a given integer is divisible by prime number without performing actual division . these rules are fundamental tools in elementary number theory and have a practical application in arithmetic , problem solving and computational mathematics .In this paper we present a rigorous proof behind for the divisibility rule corresponding to selected prime numbers ,namely 2, 3 ,7 and 11

Keywords: Prime number, Divisibility, Congruence relation ,Divisor, modular arithmetic

1 Introduction to Number Theory

Number theory is about integers and their properties.

We will start with the basic principles of

*Divisibility,

*Greatest common divisors,

*Least common multiples, and

*Modular arithmetic

and look at some relevant algorithms.

Division

If a and b are integers with a ≠ 0, we say that a divides b if there is an integer c so that b = ac.

When a divides b we say that a is a factor of b and that b is a multiple of a.

The notation a | b means that a divides b.

We write a X b when a does not divide b

Divisibility Theorems

For integers a, b, and c it is true that if a | b and a | c, then a | (b + c)

Prime A positive integer p greater than 1 is called prime if the only positive factors of p are 1 and p. A positive integer that is greater than 1 and is not prime is called composite.

The fundamental theorem of arithmetic: Every positive integer can be written uniquely as the product of primes, where the prime factors are written in order of increasing size.

Division Algorithm Let a be an integer and d a positive integer. Then there are unique integers q and r, with 0<=r < d such that a = dq + r

Relatively Prime Integers Two integers a and b are relatively prime if gcd(a, b) = 1.

Modular Arithmetic Let a be an integer and m be a positive integer. We denote by a mod m the remainder when a is divided by m.

Examples: 7 mod 3=1, 4mod 2=0 ,9 mod 10=9

Congruence

Let a and b be integers and m be a positive integer. We say that a is congruent to b modulo m if m divides a - b

We use the notation a≡b (mod m) to indicate that a is congruent to b modulo m.

In other words: a≡ b (mod m) if and only if a mod m = b mod m.

Examples:

Is it true that 46 ≡68 (mod 11)?

Yes, because 11| (46 - 68)

For which integers z is it true that z ≡12 (mod 10)?

It is true for any zє (..., - 28 ,-18,-8,2,12,22,32,...)

Theorem: Let m be a positive integer. The integers a and b are congruent modulo m if and only if there is an integer k such that a = b + km

Theorem: Let m be a positive integer.

If a ≡ b (mod m) and c ≡ d (mod m), then

a + c ≡ b + d (mod m) and ac≡ bd (mod m).

Representations of Integers

Let b be a positive integer greater than 1.

Then if n is a positive integer, it can be expressed uniquely in the form:

$n=a_k b^k+a_{k-1}b^{k-1}+\ldots+a_1 b+a_0$

where k is a nonnegative integer, $a_0, a_1, \ldots a_k$ are nonnegative integers less than b,and a≠0

2. Proof of test for divisibility by prime number

2.1 Divisibility test for 2 in [1]

Proof behind this test as follows

we know 10 divide by 2 has remainder 0 so that 10 ≡ 0(mod 2) then

$10^m \equiv 0^m$ (mod 2) for m=1,2,3…

Hence $Z \equiv a_0 + a_1.0 + a_2.0 + ... + a_k.0 \equiv a_0$ (mod 2)

Hence Z is divisible by 2 iff last digit a_0 is divisible by 2

2.2 Divisibility test for 3in [1]

Proof behind this test as follows

we know 10 divide by 3 has remainder 1so that $10 \equiv 1$(mod 3) then

$10^m \equiv 1^m$ (mod 3) for m=1,2,3…

Hence $Z \equiv a_0 + a_1.1 + a_2.1 + ... + a_k.1 \equiv a_0 + a_1 + a_2 + ... + a_k$ (mod 3)

Hence Z is divisible by 3 if and only if sum of its digits is divisible by 3

2.3 Divisibility test for 11

Proof behind this test as follows

we know $10 \equiv -1$(mod 11) then

$10^m \equiv (-1)^m$ (mod 11) for m=1,2,3…

Hence $Z \equiv a_0 + a_1.(-1) + a_2.(-1)^2 + ... + a_k.(-1)^k \equiv a_0 - a_1 + a_2 - a_3 + ... + a_k(-1)^k$ (mod 11)

Hence Z is divisible by 11 iff alternating sum of its digits $a_0 - a_1 + a_2 + ... + a_k.(-1)^k$ is divisiblie by 11

2.4 Divisibility test for 7

We know the divisibility rule for 7 in [1] Double the last digit and subtract it from the remaining leading truncated number. If the result is divisible by 7, then the original number is divisible by 7

Here we give the proof for that

Applying the divisibility rule for 7 to 100a + 10b + c is divisible by 7 iff 10a + b - 2c is divisible by 7

Proof-

Assume that 10a + b - 2c is divisible by 7

Show that 100a + 10b + c is divisible by 7

We have 10a + b - 2c = 7d

100a + 10b - 20c = 70d multiplied by 10)

100a + 10b =70d+20c

100a + 10b + c = 70d + 21c

100a + 10b + c = 7(10d + 3c)

We know R.H.S. is divisible by 7 thus L.H.S.

Hence proved

We must also show the convers is true

Assume that 100a + 10b + c is divisible by 7

Show that 10a + b - 2c is divisible by 7

We have 100a + 10b + c = 7d

100a + 10b = 7d - c (Subtract c from each side)

10a+b = 7d + (- c) / 10 (Divide each side by 10)

10a + b - 2c -=7d-c/10- 2c (Subtract 2c from each side)

10a+b-2c-= 7d - 21c / 10

10a+b-2c-= 7(d - 3c) / 10

We know R.H.S. is divisible by 7 thus L.H.S.

Hence proved

This divisibility test procedure works similarly for 13,17,19,37 each has it own subtraction multiplier

Conclusion

The study of divisibility rule for prime number reveals the deep connection between number representation and modular arithmetic . Each rule and their proof behind it appearing as a simple test of digit is firmly rooted in the positional value system of integers . by applying modular properties we that divisibility rule are not arbitrary tricks but logical consequence of number theory . these proofs not only justify the validity of the rule but also highlight the elegance of mathematics in reducing complex calculation into simple. By demonstrating these proofs ,the paper not only validate the underlying rules but also highlights their theoretical importance and relevance to the teaching of the elementary number theory .

References

[1] J.N.Salunke, N.S.Bayaskar, Divisibility test by some prime number, National conference on recent trends in mathematics (2009)

[2]M.Ahuja&J.Bruening, A survey of divisibility test with a historical perspective, Bull. Malaysian Math. Soc. (Second series)22(1999)35-43

[3]Robert H.Prior Divisibility Rule For 7(And 13, And...) March 4, 2005

[4] LE. Dickson, History of the Theory of Numbers, Vol.1, Chelsea, New York, 1971.